

Binomial Expansion

$$(1+x)^n = 1 + \frac{n}{1}x + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 \dots$$

n must be rational and not positive integer
 $|x| < 1$

Questions

Rewrite as approximations using the binomial expansion as far as the x^4 term:

1) $(1+x)^{-5}$

3) $(4+x)^{-5}$

2) $(1+2x)^{-5}$

4) $(4+12x)^{-5}$

Rewrite as approximations using the binomial expansion as far as the x^3 term:

5) $(1+x)^{\frac{1}{2}}$

7) $(4+x)^{\frac{1}{2}}$

6) $(1+2x)^{\frac{1}{2}}$

8) $(4+12x)^{\frac{1}{2}}$

9) $(1+x)^{-3} + (2+3x)^{-3}$

Extra Practice

As far as the x^4 term

10) $(1+x)^{-6}$

11) $(1+2x)^{-6}$

12) $(4+3x)^{-5}$

As far as the x^3 term

13) $(1+x)^{\frac{1}{4}}$

14) $(1+2x)^{\frac{1}{4}}$

15) $(4+12x)^{\frac{1}{4}}$

16) $(4+x)^{\frac{1}{4}}$

Application

Show that $\left(1 + \frac{x}{25}\right)^{\frac{1}{2}} = 1 + \frac{x}{50} - \frac{x^2}{5000} + \frac{x^3}{250000} - \dots$

By substituting $x=1$ into the expression above, deduce that $\sqrt{26} \approx 5.09902$

Answers

Rewrite as approximations using the binomial expansion as far as the x^4 term:

$$1) (1+x)^{-5} = 1 - 5x + 15x^2 - 35x^3 + 70x^4$$

$$2) (1+2x)^{-5} = 1 - 10x + 60x^2 - 280x^3 + 1120x^4$$

$$3) (4+x)^{-5} = 4^{-5} \left(1 + \frac{x}{4}\right)^{-5} = \frac{1}{1024} - \frac{5}{4096}x + \frac{15}{16384}x^2 - \frac{35}{65536}x^3 + \frac{35}{131072}x^4$$

$$4) (4+12x)^{-5} = 4^{-5} (1+3x)^{-5} = \frac{1}{1024} - \frac{15}{1024}x + \frac{135}{1024}x^2 - \frac{945}{1024}x^3 + \frac{2835}{512}x^4$$

Rewrite as approximations using the binomial expansion as far as the x^3 term:

$$5) (1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$$

$$6) (1+2x)^{\frac{1}{2}} = 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3$$

$$7) (4+x)^{\frac{1}{2}} = 4^{\frac{1}{2}} \left[\left(1 + \frac{x}{4}\right)^{\frac{1}{2}} \right] = 2 + \frac{1}{4}x - \frac{1}{64}x^2 + \frac{1}{512}x^3$$

$$8) (4+12x)^{\frac{1}{2}} = 4^{\frac{1}{2}} \left[(1+3x)^{\frac{1}{2}} \right] = 2 + 3x - \frac{9}{4}x^2 + \frac{27}{8}x^3$$

$$9) (1+x)^{-3} + (2+3x)^{-3} = \frac{9}{8} - \frac{57}{16}x + \frac{123}{16}x^2 - \frac{455}{32}x^3$$

Extra practice:

$$10) (1+x)^{-6} = 1 - 6x + 21x^2 - 56x^3 + 126x^4$$

$$11) (1+2x)^{-6} = 1 - 12x + 84x^2 - 448x^3 + 2016x^4$$

$$12) (4+3x)^{-5} = 4^{-5} \left[\left(1 + \frac{3}{4}x\right)^{-5} \right] = \frac{1}{1024} - \frac{15}{4096}x + \frac{135}{16384}x^2 - \frac{945}{65536}x^3 + \frac{2835}{262144}x^4$$

$$13) (1+x)^{\frac{1}{4}} = 1 + \frac{1}{4}x - \frac{3}{32}x^2 + \frac{7}{128}x^3$$

$$14) (1+2x)^{\frac{1}{4}} = 1 + \frac{1}{2}x - \frac{3}{8}x^2 + \frac{7}{16}x^3$$

$$15) (4+12x)^{\frac{1}{4}} = \sqrt{2} + \frac{3\sqrt{2}}{4}x - \frac{27\sqrt{2}}{32}x^2 + \frac{189\sqrt{2}}{128}x^3$$

$$16) (4+x)^{\frac{1}{4}} = \sqrt{2} + \frac{\sqrt{2}}{16}x - \frac{3\sqrt{2}}{512}x^2 + \frac{7\sqrt{2}}{8192}x^3$$